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The project had four major goals. The first was the development of distributed parameter models of the transient behavior of some or all of the state and internal variables describing the motion of multiple-link *flexible structures*. The structures under consideration consist of finitely many interconnected flexible elements such as strings, beams, plates and shells or combinations thereof and are representative of trusses, frames, robot arms, solar panels, antennae, deformable mirrors, etc. The second goal of the project was to provide rigorous mathematical analyses of the resulting models. Its third goal was to develop control- theoretic properties of, and control strategies for, multiple-link flexible structures based on the control-theoretic properties of the models. The fourth emphasis was on model validation and illustration with the aid of extensive numerical simulations of the predictive capabilities of the mathematical models.

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Final Technical Report
AFOSR grant F49620-92-J-0031
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PI: John E. Lagnese

This report describes the accomplishments achieved in the research project titled "Modeling, Analysis and Control of Multiple-Link Flexible Structures", supported under the above referenced research grant.

The project had four major goals. The first was the development of distributed parameter models of the transient behavior of some or all of the state and internal variables describing the motion of multiple-link *flexible structures*. The structures under consideration consist of finitely many interconnected flexible elements such as strings, beams, plates and shells or combinations thereof and are representative of trusses, frames, robot arms, solar panels, antennae, deformable mirrors, etc. The second goal of the project was to provide rigorous mathematical analyses of the resulting models. Its third goal was to develop control- theoretic properties of, and control strategies for, multiple-link flexible structures based on the control-theoretic properties of the models. The fourth emphasis was on model validation and illustration with the aid of extensive numerical simulations of the predictive capabilities of the mathematical models. Essentially all goals of the project were met.

Initial emphasis of the research was placed on the development of distributed parameter models and of a comprehensive theory of control of networks of one-dimensional elements, such as trusses and frames. Models describing the dynamic behavior of a 3-dimensional networks of thin, nonlinear thermoelastic beams were developed and tested by numerical simulations. The beams may be initially curved and twisted. In particular, the geometric and kinematic conditions which must hold at a junction where two or more beams are joined have been explicated in several situations of interest, including rigid joints and pinned joints. The very complicated interaction and transmission of elastic effects such as bending, torsion and axial deformation through the junction regions are clearly seen in the numerical simulations. A careful study of the nature of the spectrum of the such structures was carried out, and led to an explicit description of the spectrum and eigenfunctions as well as to techniques for their computation. A comprehensive theory of control and stabilization for such structures was developed. Various anomalies which may occur, such as the possibility of uncontrollable and unstabilizable modes, were pointed out and the underlying causes for such anomalies were clarified, as well as possible remedies.

Attention was next focused on the development of models of structures consisting of interconnected two-dimensional elements, such as thin plates or shells,

or combinations of one- and two-dimensional elements, such as linked beams and plates, and their associated control-theoretic properties. These problems are much more subtle than their one-dimensional analogs, beginning with the notion a joint, or junction, between two-dimensional elements, and new complicating phenomena arise in the analysis, such as the development of singularities (regions of high stress or where cracks occur). It turned out to be possible to give a complete controllability analysis in certain cases of interest, such as a thin elastic plate to which a thin elastic rod is rigidly attached either to its edge or to its face. However, a complete analysis of other configurations has proved to be more elusive. Nonetheless, while it cannot be claimed that all issues have been settled in either the modeling or control-theoretic aspects, substantial progress was made and the major remaining technical issues were elucidated. In addition, numerical simulations carried out on proposed models of certain configurations, such as those consisting of multiple interconnected elastic plates, lend credence to the models and to the control strategies suggested, even though the theoretical foundations are not yet completely in place.

Book

(A) Lagnese, J. E., Leugering, G. and Schmidt, E.J.P.G., "Modeling, Analysis and Control of Multi-link Flexible Structures," Birkhäuser, Boston, Mass., 1994.

Research Papers

(A) Lagnese, J. E., "Modeling and controllability of plate-beam systems," *J. Math. Systems, Estimation and Control*, 5, (1995), pp. 141–188.

(B) Lagnese, J. E., "Controllability of systems of interconnected membranes," *Discrete and Continuous Dynamical. Systems.*, 1, (1995), pp. 1–16.

(C) Lagnese, J. E., Leugering, G. and Schmidt, E.J.P.G., "On the analysis and control of hyperbolic systems associated with vibrating networks," *Proc. Royal Soc. Edinburgh*, **124**, (1994), pp. 77–104.

(D) Lagnese, J. E., Leugering, G. and Schmidt, E.J.P.G., "Modeling of dynamic networks of thin thermoelastic beams," *Math. Methods in Applied Sci.*, 16, (1993), pp. 327–358.

(E) Lagnese, J. E. and Leugering, G., "Modeling of dynamic networks of thin elastic plates," *Math. Methods in Applied Sci.*, 16, (1993), pp. 379–407.

(F) Lagnese, J. E., Leugering, G. and Schmidt, E.J.P.G., "Controllability of planar network of Timoshenko beams," *SIAM J. Control and Opt.*, 31, (1993), pp. 780–811.

(G) Lagnese, J. E., "Boundary controllability in transmission problems for thin plates," in *Differential Equations, Dynamical Systems and Control Sciences*, Marcel Dekker, New York, 1993, pp. 641–658.

(H) Lagnese, J. E., "Boundary controllability of nonlinear beams to bounded states," *Proc. Royal Soc. Edinburgh*, **119A**, (1991), pp. 63–72.

(I) Lagnese, J. E., "The Hilbert uniqueness method: A retrospective," *Lecture Notes in Control and Information Sciences*, **149**, (1991), pp. 158–181.

(J) Lagnese, J. E., "Modeling and stabilization of nonlinear plates," *International Series of Numerical Math.*, **100**, (1991), pp. 247–264.

(K) Lagnese, J. E., and Leugering, G., "Uniform stabilization of a nonlinear beam by nonlinear boundary feedback," *J. Differential Equations*, **91**, (1991), pp. 355–388.

(L) Lagnese, J. E., "Local controllability of dynamic von Karman plates," *Control and Cybernetics*, *19*, (1991), pp. 155–168.

(M) Lagnese, J. E., "Uniform asymptotic energy estimates for solutions of the equations of dynamic plane elasticity with nonlinear dissipation at the boundary," *Nonlinear Analysis: Theory, methods and applications*, *16* (1991), pp. 35–54.

(N) Lagnese, J. E., and Leugering, G., "Uniform energy decay of a class of cantilevered nonlinear beams with nonlinear dissipation at the free end," *Lecture Notes in Pure and Applied Math.*, **133**, (1991), pp. 227–240.

Invited Presentations

(A) Department of Mathematics, Rutgers University, March, 1995.

(B) Department of Mathematics, University of Delaware, March, 1994.

(C) International Conference of Control and Estimation of Distributed Parameter Systems: Nonlinear Phenomena, Vorau, Austria, July 20, 1993.

(D) Minisymposium on Control of Partial Differential Equations, College Station, TX, October 21, 1993.

(E) Special Session on Control Systems Governed by Partial Differential Equations, AMS regional meeting, College Station, TX, October 23, 1993.

(F) SIAM Conference on Control and Its Applications, Minneapolis, MN, September 1992. (Plenary Lecture)

(G) Joint AMS, IMS, SIAM Summer Research Conference, Mount Holyoke, MA, July, 1992. (Plenary Lecture)

(H) Special Session on Function Theoretic Methods in Differential Equations, Annual meeting of the AMS, Annapolis, MD, January, 1992.

(I) IFIP Conference on Boundary Control and Boundary Variation, Sophia Antipolis, France, June, 1992.

(J) INRIA Conference on State and Frequency Domain Approaches for Infinite Dimensional Systems, Sophia Antipolis, June, 1992.

(K) Institute for Mathematics and its Applications, Minneapolis, MN, November 1992.

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